Math 10a

November 8, 2014

Probability Distributions, Expected Value, Variance

1. Consider a spinner where one third of the area is labeled with the number 1, one fourth of the area is labeled with the number 2, one third of the area is labeled with the number 3, and one twelveth of the area is labeled with the number 4.

   (a) Let X be the number showing after spinning the spinner once. What is the pmf of X? Find $E(X)$, $E(X^2)$, $\text{Var}(X)$, and $E(\sqrt{X})$.

   (b) Sketch the cdf of X.

   (c) Suppose you spin the spinner once. What is the probability that you get a number greater than 1?

   (d) Suppose you spin the spinner five times. What is the chance that 1 does not come up during those five times?

   (e) Suppose you spin the spinner five times. What is the chance that 1 comes up at least once? (hint: $1 - \text{something}$)

2. Consider a fair six sided-die.

   (a) What is the chance that you roll:

   - two 1s in two rolls of the die?
   - two 1s in three rolls of the die?
   - two 1s in four rolls of the die?
   - two 1s in five rolls of the die?

   (b) Let X be the number of 1s rolled in four rolls of die. What kind of random variable is X?

   (c) What is the pmf of X? Sketch the cdf.

   (d) Compute $E(X)$ and $E(X^3)$.

3. Recall that a Poisson random variable is one whose pmf is of the form:

$$P(X = x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

   for some λ. Random variables with Poisson distributions typically come from processes where there are an enormous amount of things each with a tiny chance of producing something: e.g. in a hunk of radioactive material there are lots of atoms ($\sim 10^{23}$), each with a tiny chance of producing an α-particle, so the number of α-particles emitted by plutonium in a 1-second interval is an example of a Poisson random variable.

   Let X be the number of α-particles emitted by a hunk of plutonium in a 1-second interval. It is a Poisson random variable with parameter $\lambda = 4$.

(a) On average, how many α-particles would you expect to see in 1-second interval? (for the a Poisson random variable, the expected value is λ)

(b) What is the chance you see no α-particles in a one-second interval? What is the chance you see 0 or 1 or 2 α-particles in a one-second interval? What is the chance you see at least 3 α-particles in a one-second interval?

4. Let X be a uniform random variable on the interval from 0 to 1 (this means you're selecting a number between 0 and 1 with all numbers given the same chance of being selected)

   (a) Sketch the pdf of X.

   (b) What is the expected value of X? What is the variance of X?

   (c) Repeat this question, but now let X be uniform on the interval from -5 to 5.

5. Let X be a continuous random variable with pdf given by

$$f(x) = \begin{cases} e^{-x} & x \geq 0 \\ 0 & x < 0 \end{cases}.$$

   What are $E(X)$ and $\mathrm{Var}(X)$?